

Kaon semileptonic decays in lattice QCD with exact chiral symmetry

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1. introduction

$K \rightarrow \pi$ decay form factors

$$\langle \pi^+(p') | V_\mu | K^0(p) \rangle = (p + p')_\mu f_+(q^2) + (p - p')_\mu f_-(q^2),$$

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2), \quad \xi(q^2) = \frac{f_-(q^2)}{f_+(q^2)}$$

- vector form factor at $q^2 = 0$: $f_+(0)$ \Leftrightarrow determination of $|V_{us}|$
- $M_{\pi, \text{sim}} \lesssim M_K$: $N_f = 3$: RBC/UKQCD, 2008, 2010, $N_f = 2$: ETM, 2009
 (see also MILC/FNAL/HPQCD, 2005; FNAL/MILC @ Lat'11)
- $\Gamma \Rightarrow |V_{us}| f_+(0) = 0.2163(5)$ (FlaviaNet, 2010) $\Leftrightarrow f_+(0)$ w/ $\lesssim 1\%$ accuracy
- other information of ME : $f_-(0)$, $\lambda_{+,0}^{\prime\prime}$ \Leftrightarrow consistency with ChPT, exp't

1. introduction

this talk

report on JLQCD's calculation of $K \rightarrow \pi$ form factors in $N_f = 2+1$ QCD

- overlap quarks \Rightarrow straightforward comparison w/ ChPT ($a=0$)
- all-to-all quark prop. \Rightarrow precise calculation of relevant meson correlators

outline

- simulation method
- extraction of form factors
- q^2 interpolation
- chiral extrapolation

2. simulation method

configurations

- $N_f = 2+1$ QCD
- Iwasaki gauge + overlap quarks + $\det[H_W^2]/\det[H_W^2 + \mu^2] \Rightarrow$ speed up, fix Q
- $a = 0.1120(5)(3)$ fm (M_Ω as input) $\Leftrightarrow O((a\Lambda_{\text{QCD}})^2) \approx 8\%$ error

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measurements of meson correlators

- 4 m_{ud} 's $\Rightarrow M_\pi \simeq 290 - 540$ MeV; $m_s = 0.080 \Leftrightarrow m_{s,\text{phys}} = 0.081$
- $16^3 \times 48$ or $24^3 \times 48$ (depending on m_{ud}) $\Rightarrow M_\pi L \gtrsim 4$
- in $Q=0$ sector \Rightarrow fixed Q effects $\propto V^{-1}$; sub-% for $M_{\{\pi,K\}}, f_{\{\pi,K\}}$
- 50 conf \times 50 HMC traj. for each (m_{ud}, m_s)

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- 50 conf \times 50 HMC traj. for each (m_{ud}, m_s)
- w/ all-to-all propagator \Rightarrow 160-240 low-modes + noise method
- twisted boundary conditions (TBCs) $\Rightarrow q^2 \in [-0.1 \text{ GeV}^2, q_{\max}^2]$
- reweighting \Rightarrow additional $m_s = 0.060$

3.1 ratio method

$$C_{V_\mu}^{PQ}(\mathbf{p}, \mathbf{p}') = \langle \mathcal{O}_Q(\mathbf{p}') V_{\mu, \text{lat}} \mathcal{O}_P^\dagger(\mathbf{p}) \rangle \sim \frac{\sqrt{Z_Q(\mathbf{p}') Z_P(\mathbf{p})}}{4E_P E_Q Z_V} \langle Q(\mathbf{p}') | V_\mu | P(\mathbf{p}) \rangle e^{-E_Q \Delta t' - E_P \Delta t}$$

$$C^P(\mathbf{p}) = \langle \mathcal{O}_P(\mathbf{p}') \mathcal{O}_P^\dagger(\mathbf{p}) \rangle \sim \frac{Z_P}{2E_P} e^{-E_Q \Delta t'} \quad (P, Q = K \text{ or } \pi)$$

ratio method (*Hashimoto et al., 1999*)

(partially) cancel Z_V , $\exp[-E_{\pi(K)}\Delta t]$, fluctuation in $C_{V_\mu}^{PQ}(\mathbf{p}, \mathbf{p}')$ and $C^P(\mathbf{p})$

$$R = \frac{C_{V_4}^{K\pi}(\mathbf{0}, \mathbf{0}) C_{V_4}^{\pi K}(\mathbf{0}, \mathbf{0})}{C_{V_4}^{KK}(\mathbf{0}, \mathbf{0}) C_{V_4}^{\pi\pi}(\mathbf{0}, \mathbf{0})} \rightarrow \frac{(M_K + M_\pi)^2}{4M_K M_\pi} f_0(q_{\max}^2)^2 \quad (q_{\max}^2 = (M_K - M_\pi)^2)$$

$$\tilde{R} = \frac{C_{V_4}^{K\pi}(\mathbf{p}, \mathbf{p}') C^K(\mathbf{0}) C^\pi(\mathbf{0})}{C_{V_4}^{K\pi}(\mathbf{0}, \mathbf{0}) C^K(\mathbf{p}) C^\pi(\mathbf{p}')} \rightarrow \left\{ 1 + \frac{E_K(\mathbf{p}) - E_\pi(\mathbf{p}')}{E_K(\mathbf{p}) + E_\pi(\mathbf{p}')} \xi(q^2) \right\} \frac{f_+(q^2)}{f_0(q_{\max}^2)}$$

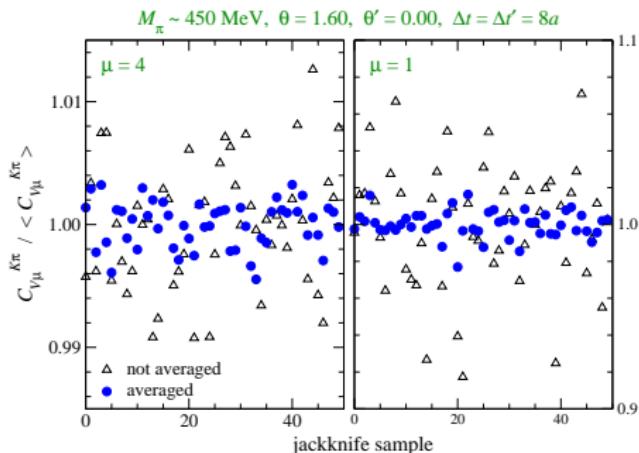
$$R_k = \frac{C_{V_k}^{K\pi}(\mathbf{p}, \mathbf{p}') C_{V_4}^{KK}(\mathbf{p}, \mathbf{p}')}{C_{V_4}^{K\pi}(\mathbf{p}, \mathbf{p}') C_{V_k}^{KK}(\mathbf{p}, \mathbf{p}')} \rightarrow \text{a function of } \xi(q^2)$$

⇒ can construct $f_+(q^2)$, $f_0(q^2)$, $\xi(q^2)$

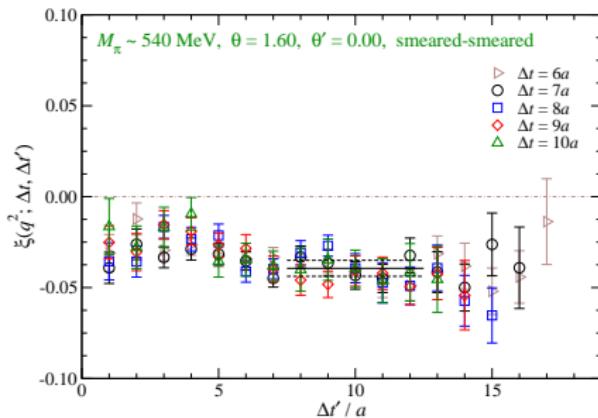
3.2 extraction of form factors : statistical accuracy

all-to-all propagator \Rightarrow can improve stat. accuracy

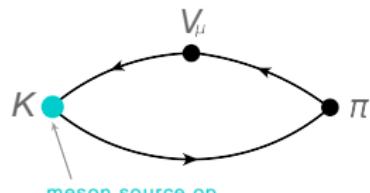
$C_{V_\mu}^{K\pi}$ at each jackknife sample



$\xi(q^2; \Delta t, \Delta t')$ vs Δt



- all-to-all \Rightarrow average over location of meson source op.
 \Leftrightarrow a factor of 3 (4) improvement for $\mu = 4$ (1)
- clear signal for $\xi(q^2)$: 10 – 30% (comparison w/ exp't (later))
- $f_{\{+,0\}}(q^2)$: statistical accuracy sub-% level



3.2 extraction of form factors : statistical accuracy

reweighting \Rightarrow can impair stat. accuracy

$$\langle C_{V_\mu}^{PQ} \rangle_{m'_s} = \langle C_{V_\mu}^{PQ} \tilde{w}(m'_s, m_s) \rangle_{m_s}$$

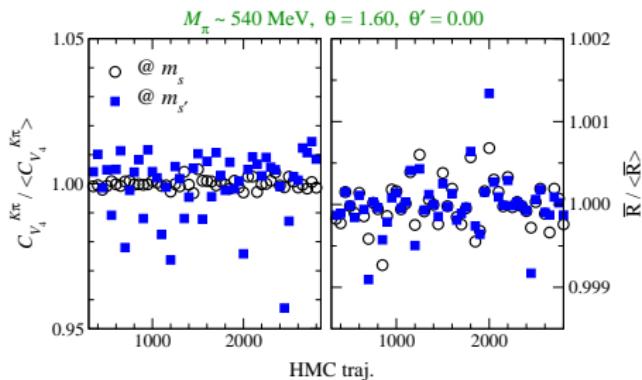
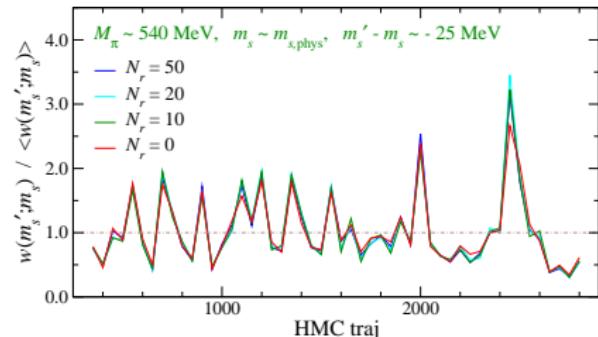
$$\tilde{w}(m'_s, m_s) = \frac{w(m'_s, m_s)}{\langle w(m'_s, m_s) \rangle_{m_s}}$$

$$w(m'_s, m_s) = \det[D(m'_s)] / \det[D(m_s)]$$

$$= \frac{\prod_{k=1}^{N_e} \lambda_k(m'_s)}{\prod_{k=1}^{N_e} \lambda_k(m_s)} \times \frac{1}{N_r} \sum_{r=1}^{N_r} e^{-\frac{1}{2} \xi_r^\dagger \frac{D(m'_s)}{D(m_s)} \xi_r}$$

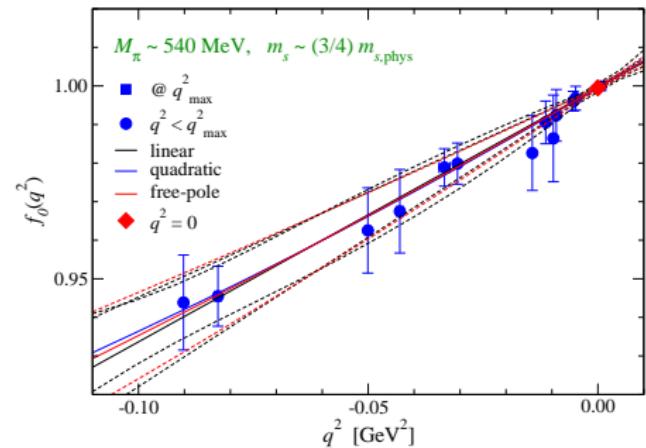
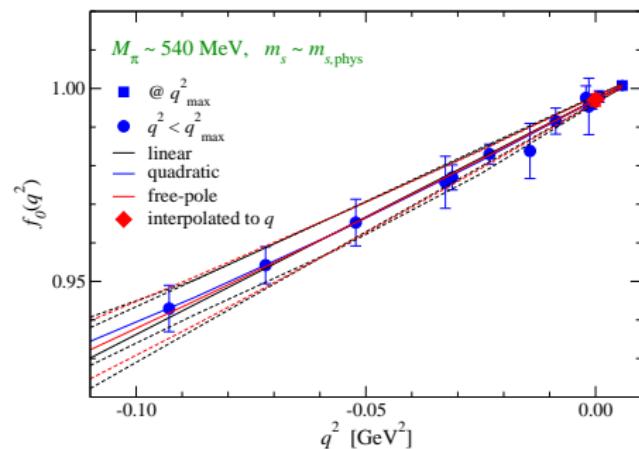
- $24^3(16^3) \times 48, \approx 200$ low-modes
 \Rightarrow do not need large N_r
- largely increase $\Delta[C_{V_\mu}^{PQ}]$
- at most $\times 2$ for ratios

all-to-all + ratio + reweight $\Rightarrow \Delta[f_{+,0}(q^2)] \lesssim 1.5\%, \quad \Delta[\xi(q^2)] \sim 20 - 40\%$



4. q^2 -dependence

$f_0(q^2)$ vs q^2

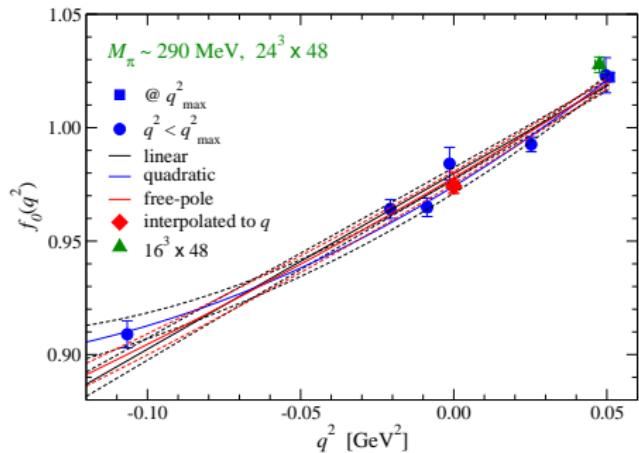


- reweighting \Rightarrow slightly larger error
- small curvature in simulated region of q^2
 $\Leftrightarrow M_{\text{pole}} = 1.2 - 1.3$ GeV @ $m_{q,\text{phys}}$ (PDG, 2010) $\Rightarrow q^4/M_{\text{pole}}^4 \lesssim 0.6\%$
- well described by any of

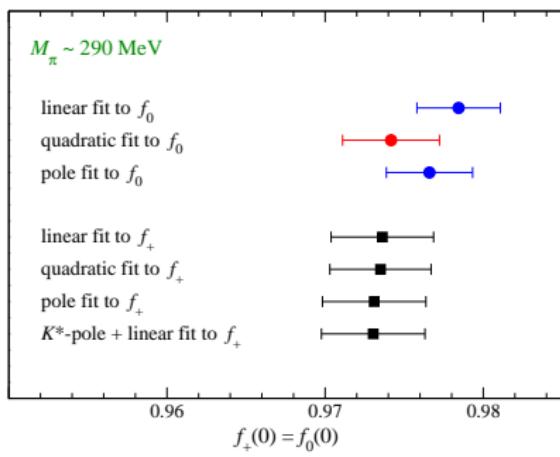
$$f_0(q^2) = f_0(0)(1 + a_1 q^2), \quad f_0(0)(1 + a_1 q^2 + a_2 q^4), \quad \frac{f_0(0)}{1 - q^2/M_{\text{pole}}^2}$$

4. q^2 -dependence

$f_0(q^2)$ vs q^2



fit results for $f_+(0)$



- $f_+(0)$ ($= f_0(0)$), $df_{+,0}(0)/dq^2|_{q^2=0}$: stable against choice of fit form
- small finite volume effects \Leftrightarrow lattice boundaries + fixed Q
- $16^3 \times 48 \Leftrightarrow 24^3 \times 48$: 0.4–0.7% ($\lesssim 2.2\text{-}2.5\sigma$) \Rightarrow even smaller on $24^3 \times 48$ (?)
- this talk : quadratic fit $\Rightarrow f_+(0)$, $df_+(q^2)/dq^2|_{q^2=0}$

5.1 chiral fit : $f_+(0)$

chiral expansions (cf. JLQCD, 2008)

$$f_+(0) = 1 + f_2 + f_4 + \dots = 1 + \textcolor{blue}{f}_2 + \Delta f$$

- $x_{\pi(K)} \equiv M_{\pi(K)}^2 / (4\pi \textcolor{blue}{F}_0)^2$ (“x-expansion”)

small $F_0 = 52.5(5.1)$ MeV $\ll F_\pi \Rightarrow$ enhanced chiral corrections $\mathcal{O}_{2n} = (M_\pi/F_0)^{2n}$

- $\xi_{\pi(K)} \equiv M_{\pi(K)}^2 / (4\pi \textcolor{blue}{F}_\pi)^2$ (“ξ-expansion”)

better convergence for $M_{\{\pi, K\}}$, $F_{\{\pi, K\}}$ (JLQCD, 2008 ($N_f = 2$) ; 2009 ($N_f = 2 + 1$))

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fitting w/ ξ-expansion

- NLO (Gasser-Leutwyler, 1985)

Ademollo-Gatto $\Rightarrow \textcolor{blue}{f}_2(F_\pi, M_{\{\pi, K, \eta\}})$

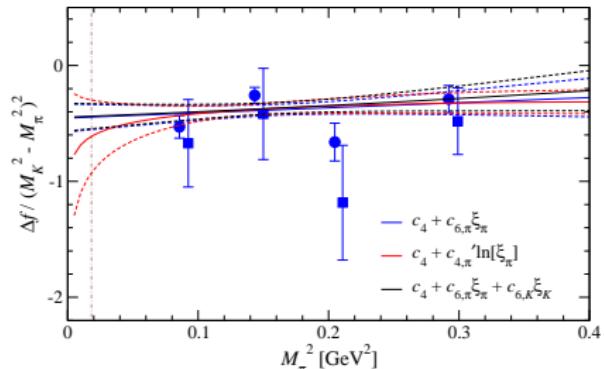
- NNLO and higher orders

(Post-Schilcher, 2001; Bijnens-Talavera, 2003)

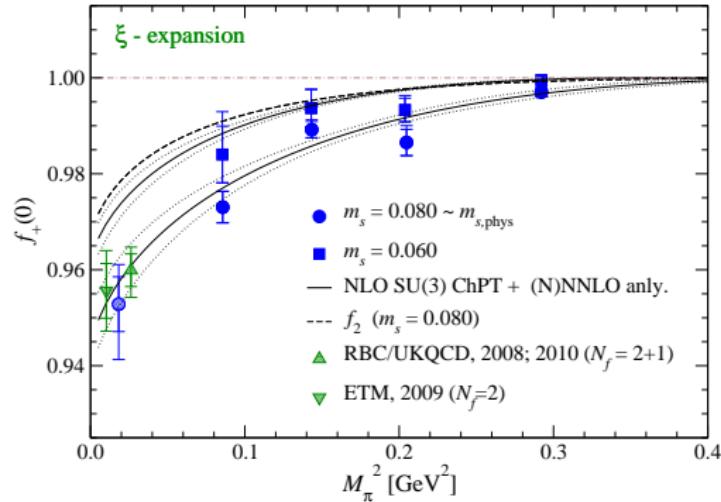
$\Delta f(F_\pi, \{L_i\}, \{C_i\}, M_{\{\pi, K, \eta\}})$

- modeling Δf (as in previous studies)

$$\Delta f = (M_K^2 - M_\pi^2)^2 (\textcolor{red}{c}_4 + \textcolor{red}{c}'_4 \ln[\xi_\pi], \textcolor{red}{c}''_4 \ln^2[\xi_\pi], c_{6,\pi} \xi_\pi, c_{6,K} \xi_K)$$



5.1 chiral fit : $f_+(0)$

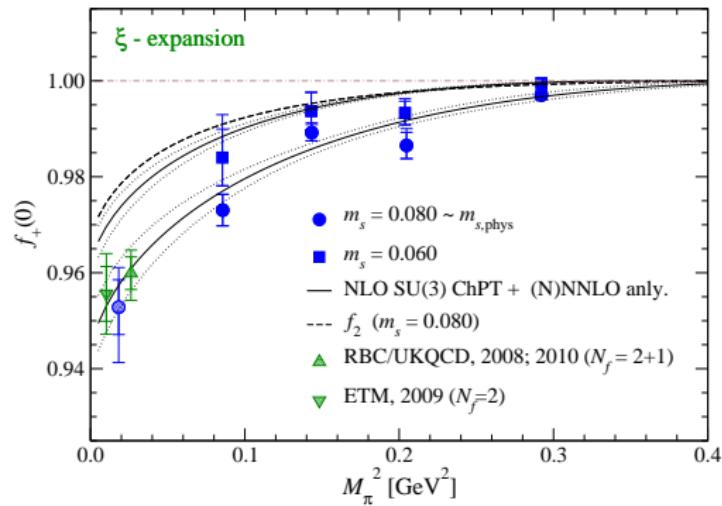


fit results

- test parameterizations of Δf
 - $c_4 + c_{6,\pi} \xi_\pi \Rightarrow$ central value
 - c_4
 - $c_4 + c_{6,\pi} \xi_\pi + c_{6,K} \xi_K$
 - $c_4 + c'_{4,\pi} \ln[\xi_\pi]$ (ill-determined c_X)
 - fit excluding largest m_{ud}
- assume $O((a\Lambda)^2)$ error in $f_2 + \Delta f$

$$f_+(0) = 0.953(6)_{\text{stat}} \left(\begin{array}{c} +4 \\ -9 \end{array} \right)_{\text{chiral}} (4)_{a \neq 0}$$

5.1 chiral fit : $f_+(0)$



fit results

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CKM unitarity

$$|V_{us}|f_+(0) = 0.2163(5) \quad (\text{FlaviaNet, 2010}) \quad \Rightarrow \quad |V_{us}| = 0.2270(+20/-27)$$

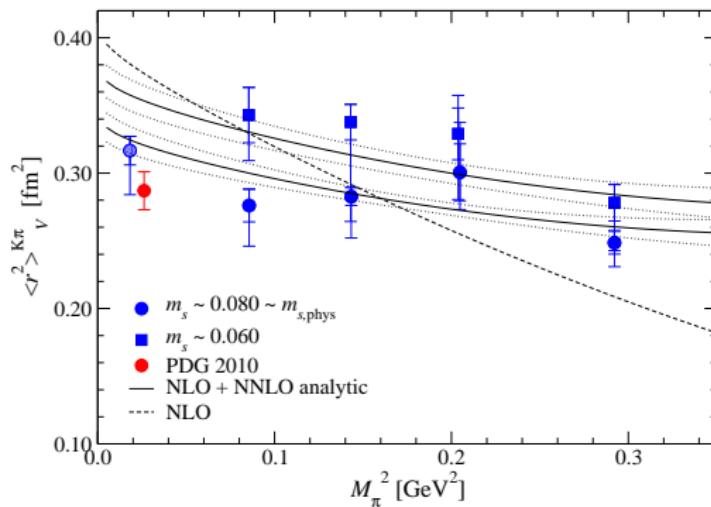
$$|V_{ud}| = 0.9743(2), \quad |V_{ub}| \sim 10^{-3} \quad (\text{ICHEP'10})$$

$$\Delta_{\text{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0.0007(+9/-13)$$

5.2 chiral fit : $\langle r^2 \rangle_V^{K\pi}$

slope of $f_+(q^2)$

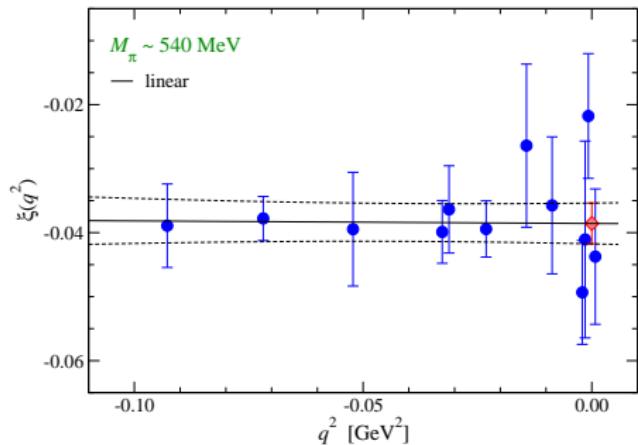
- $f_+(q^2) = f_+(0) \{1 + (1/6) \langle r^2 \rangle_V^{K\pi} q^2 + \dots\}$
- SU(3) NLO ChPT (Gasser-Leutwyler, 1985) + NNLO analy., ξ -expansion,
 $\langle r^2 \rangle_V^{K\pi} = 12 L_9^r / F_\pi^2 - (1/F_\pi^2)$ "logs" + $c_\pi \xi_\pi + c_K \xi_K$



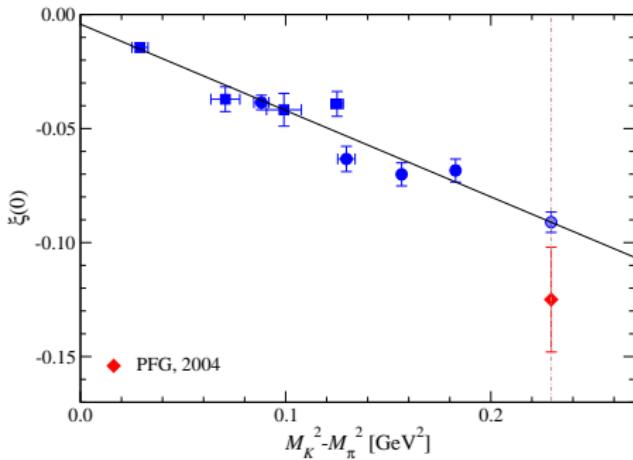
- significant NNLO @ $m_{ud(s),\text{sim}}$
 \Rightarrow milder m_{ud} -dependence
 $\Leftrightarrow \langle r^2 \rangle_V^\pi, \langle r^2 \rangle_V^K$ (JLQCD, $N_f = 2, 3$)
- $L_9^r(M_\rho)$: consistent w/ pheno.
 - $L_9^r = 6.2(0.5)(2.7) \times 10^{-3}$
 - pheno. : $5.91(4) \times 10^{-3}$
 $(Bijnens, 2007, \xi\text{-expansion})$
- $\langle r^2 \rangle_V^{K\pi}$: consistent w/ exp't

5.3 chiral fit : $\xi(0) = f_-(0)/f_+(0)$

$\xi(q^2)$ vs q^2



$\xi(0)$ vs $M_K^2 - M_\pi^2$



- mild q^2 dependence w/ our statistical accuracy

$$cf. \quad f_{-, \text{analy}}(q^2) = (M_K^2 - M_\pi^2) (4L_5^r/F_\pi^2 - 2L_9^r/F_\pi^2) \quad (\text{Bijnens-Talavera 2003})$$

- (for simplicity) fit form linear in $(M_K^2 - M_\pi^2)$: $\xi(0) = c_0 + c_1(M_K^2 - M_\pi^2)$

- vanish in SU(3) limit as expected : $c_0 = -0.0017(9)$
- consistent w/ experiment : $\xi(0) = -0.125(23)$ (PDG 2004, K_{l3}^+)
- one-loop chiral log.s (and two-loops?) should be included (Bijnens-Talavera 2003)

6. summary

kaon semileptonic form factors in $N_f = 2 + 1$ QCD with overlap quarks

- techniques

- all-to-all propagators \Rightarrow precise determination of $f_{+,0}(q^2)$
- TBCs \Rightarrow precise determination of $f_+(0)$, $\xi(0)$, $\langle r^2 \rangle_{V,S}^{K\pi}$
- reweighting \Rightarrow additional m_s

- chiral fits

- $f_+(0) = 0.953(6)_{\text{stat}} ({}^{+4}_{-9})_{\text{chiral}} (4)_{a \neq 0}$, $|V_{us}| = 0.2270 ({}^{+17}_{-26})$,
- $$\Delta_{\text{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 7 ({}^{+9}_{-13}) \times 10^{-4}$$
- $\langle r^2 \rangle_V^{K\pi}$, $\xi(0)$: reasonably consistent w/ exp't

- future directions

- further refinements \Leftrightarrow more rigorous treatment of NNLO \Leftrightarrow overlap quarks
- π , K EM form factors \Leftrightarrow test of ChPT
- D , B meson decays \Leftrightarrow flavor physics

7.1 chiral fit : $f_+(0)$

F_0

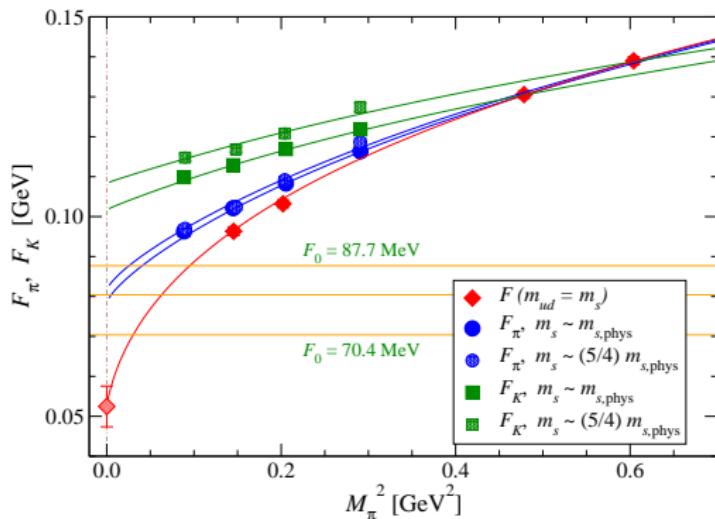
- phenomenological estimates

(Bijnens @ Lattice'07; $\mu = 0.77$ GeV)

| | | | |
|--------------|------------|--------------|--------------|
| $10^3 L_4^r$ | $\equiv 0$ | $\equiv 0.5$ | $\equiv 0.2$ |
| $10^3 L_6^r$ | $\equiv 0$ | $\equiv 0.1$ | $\equiv 0$ |
| F_0 [MeV] | 87.7 | 70.4 | 80.4 |

- JLQCD's analysis (Noaki @ Lattice'10)

- NNLO fit to $M_{\{\pi, K\}}$, $F_{\{\pi, K\}}$
- chiral expansion w/ M_π^2/F_π^2 instead of M_π^2/F_0
⇒ better convergence
- single lattice spacing ~ 0.11 fm
- $F_0 = 52.5(5.1)$ MeV



small $F_0 \Rightarrow$ enhance chiral corrections : NLO $\propto 1/F_0^2$; NNLO $\propto 1/F_0^4$